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F.E Sem-I (Revised Course 2019-2020)  
EXAMINATION NOV/DEC 2019  
Mathematics-I

[Duration : Three Hours]

[Total Marks : 100]

Instructions:-

1. Answer five questions. At least two from part-A, two from part –B and one from part – C.
2. Assume suitable data, if necessary.
3. Figures to right indicate full marks.

**Part A**

Answer any two questions of the following.

Q.1 a) Evaluate  $\int_0^{\infty} x^{\frac{3}{2}} e^{-x^2} dx$  (4 marks)

b) Use Taylor's theorem to expand  $f(x) = \sqrt{1+x+2x^2}$  in powers of  $(x-1)$  using Taylor's series. (4 marks)

c) Test the convergence of the following series (12 marks)

i.  $\sum_{n=1}^{\infty} n \sin^2\left(\frac{1}{n}\right)$

ii.  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$

iii.  $\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \frac{5}{26} - \dots$

Q.2 a) If  $y = (x + \sqrt{p^2 + x^2})^2$  then prove that (6 Marks)

$$(p^2 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - 4)y_n = 0$$

b) Find the interval of convergence of the following series (8 Marks)

$$\sum_{n=1}^{\infty} \frac{3^n x^n}{\sqrt{n}}$$

c) Evaluate  $\int_5^8 \sqrt[3]{(x-5)(8-x)} dx$  (6 Marks)

Q.3 a) Evaluate (12 Marks)

i)  $\lim_{x \rightarrow 0} (\cos \sqrt{x})^{\frac{1}{x}}$

ii)  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{\cot \frac{\pi x}{2}}$

iii)  $\lim_{y \rightarrow -4} \frac{\sin(\pi y)}{y^2 - 16}$

b) Evaluate  $\int_0^{\infty} \frac{x^5 + x^7}{(1+x)^{14}} dx$

c) Find the expansion of  $e^{\cos x}$  up to  $x^5$

(4 Marks)

(4 Marks)

**Part B**

Solve the two questions of the following

Q.4 a) Solve the following differential equations

i.  $\sec x dy = (y + \sin x) dx$

ii.  $(y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0$

iii.  $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

(12 Marks)

b) If  $u(x, y, z) = \log(\tan x + \tan y + \tan z)$ , find the value of

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z}$$

(8 Marks)

Q.5 a) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube use Lagrange's method of undetermined multipliers.

(08 Marks)

b) If  $u = \tan^{-1}(x^2 + 2y^2)$  then evaluate

(06 Marks)

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

c) Solve

$$\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$$

(06 Marks)

Q.6 a) Find the values of  $x$  and  $y$  for which  $x^2 + y^2 + 6x = 12$  has a minimum value and find this minimum value.

(06 Marks)

b) Verify Euler's Theorem for

(08 Marks)

$$u = x^2 \tan^{-1} \left( \frac{y}{x} \right) - y^2 \tan^{-1} \left( \frac{x}{y} \right)$$

c) Solve  $(y - 2x^3) dx - x(1 - xy) dy = 0$

(06 Marks)

**Part C**

Answer any one question of the following.

Q.7 a) Prove that  $\beta(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$

(08 Marks)

b) If  $u = \sin^{-1} \left( \frac{x^4 + y^4}{1 + x^6 + y^6} \right)$ , find  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$

(07 Marks)

- c) Evaluate  $\lim_{x \rightarrow a} \frac{\log(x-a)}{\log(e^x - e^a)}$  (05 Marks)

- Q.8 a) Define absolutely convergent and conditionally convergent series and test the absolute convergence and conditional convergence of the following series. (08 Marks)

$$1 - \frac{1}{5} + \frac{1}{9} - \frac{1}{13} + \dots$$

- b) Prove that  $\log \tan \left( \frac{\pi}{4} + x \right) = 2x + \frac{4x^3}{3} + \dots$  (07 Marks)

- c) Solve  $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$  (05 Marks)