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S.E.(Mining) (Sem-III) (Revised Course 2016-2017) EXAMINATION MAY/JUNE 2019
Engineering Mathematics-III

[Duration : Three Hours]

[Max.Marks : 100]

Instructions:

- 1) Attempt five questions, any two questions each from PART-A and PART-B and one from PART-C.
- 2) Assume suitable data, if necessary.
- 3) Figures to the right indicate full marks.

PART - A

Answer any TWO questions from the following:

2×20=40

- Q.1**
- a) Define a symmetric and a skew symmetric matrix. Give one example of each. Show that the diagonal elements of a skew symmetric matrix are zeros. **06**
 - b) Define rank of a matrix. By reducing to normal form, find the rank of the given matrix. **08**

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 2 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \\ 1 & 3 & -1 & -3 \end{bmatrix}$$

- c) Solve for $f(x)$, the following integral equation **06**

$$\int_0^{\infty} f(x) \cos \lambda x \, dx = \begin{cases} 1 - \lambda & \text{if } 0 < \lambda < 1 \\ 0 & \text{if } \lambda > 1 \end{cases}$$

- Q.2**
- a) Test the consistency of the following equations and solve them if consistent. **08**

$$\begin{aligned} 3x + 3y + 2z &= 1 \\ x + 2y &= 4 \\ 10y + 3z &= -2 \\ 2x - 3y - z &= 5 \end{aligned}$$

- b) Find the Eigen values and corresponding Eigen vectors of the matrix A^{-1} **08**

$$\text{where } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

- c) If $F\{f(x)\} = F(\lambda)$, then show that $F\{xf(x)\} = -i \frac{d}{d\lambda} F(\lambda)$ **04**

- Q.3 a) Find the Fourier transform of
 $f(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$

Hence evaluate the integral $\int_0^\infty \frac{\sin^4 x}{x^4} dx$

- b) Find the Fourier series of the function
 $f(x) = \begin{cases} \pi + 2x & \text{if } -\pi < x < 0 \\ \pi - 2x & \text{if } 0 < x < \pi. \end{cases}$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

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PART-B

Answer any TWO questions from the following:

2×20=40

- Q.4 a) Find the Laplace transform of

(i) $e^t \sin t \cos 2t$ (ii) $t \cos^2 t$

08

- b) Using Laplace transforms, solve the integral equation

$$y(t) = 1 + \int_0^t y(u) \sin(t-u) du$$

06

- c) Solve the following partial differential equation using various separable method.

06

$$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}; u(0, y) = 8e^{-3y}$$

- Q.5 a) Stating all assumptions, derive the one dimensional heat equation.

08

- b) Find the inverse Laplace transform of

08

(i) $\log \left[\frac{s^2+1}{s^2-4} \right]$ (ii) $\frac{1}{(s-2)(s+4)}$

- c) Define unit step function and find its Laplace transform.

04

- Q.6 a) Using convolution theorem, find the inverse Laplace transform of $\frac{1}{(s+1)(s^2+1)}$

06

- b) Using variable separable method, derive the solution of one dimensional heat equation.

08

- c) Let $L\{f(t)\} = F(s)$, then prove that

06

i) $L(e^{at} f(t)) = F(s-a)$

ii) $L\{f'(t)\} = sF(s) - f(0)$

PART – C

Answer any **ONE** question from the following:

- Q.7 a) Verify Cayley Hamilton theorem for the following matrix.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Hence find the matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

- b) Find the half range Fourier cosine series of $f(x) = 2x - x^2, 0 < x < 2$.

c) Find e^A , where $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

- Q.8 a) If $f(t)$ is a continuous periodic function having period T , then prove that

$$L\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$$

- b) Using Laplace transforms, evaluate the integral $\int_0^\infty e^{-t} t \sin 2t dt$.

- c) Stating all the assumptions, derive the one-dimensional wave equation.

1×20=20

08

06

06

06

04

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