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S.E.(Mining) (Semester- III) (Revised Course 2007-08) EXAMINATION MAY/JUNE 2019

Engineering Mathematics - III

[Duration : Three Hours]

[Max. Marks : 100]

Please check whether you have got the right question paper.

Instruction:

1. Answer any five questions with at least one from each Module.
2. Assume suitable data if necessary.

## MODULE-I

- Q.1 a) Prove the following 06
- i) The determinant of Hermitian matrix is real.
  - ii) If A is a nonsingular square matrix of order n then  $\text{adj. adj.} A = |A|^{n-2}$
- b) Reduce the following matrix in the normal form and find its rank 08
- $$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$
- c) Test the consistency for the following equations and solve them if consistent. 06
- $$2x + 3y + 4z = 11, \quad x + 5y + 7z = 15, \quad 3x + 11y + 13z = 25$$
- Q.2 a) Determine the linear dependence or independence of the following vectors and find the relation between them if they are dependent. 06
- $$(3, 1, -4), \quad (2, 2, -3), \quad (0, -4, 1)$$
- b) Diagonalize the matrix  $A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$  08
- c) Verify Cayley – Hamilton theorem and hence find  $A^{-1}$  06
- $$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

## MODULE-II

- Q.3 a) Find Fourier series for the following functions. 10
- $$f(x) = -x, \quad -\pi \leq x \leq 0$$
- $$= x, \quad 0 \leq x \leq \pi$$

- b) Obtain Fourier cosine series for  $x(\pi - x)$  in  $0 \leq x \leq \pi$  10  
 Hence obtain  $\sum_{n=1}^{\infty} \frac{1}{n^4}$

- Q.4 a) Find Fourier Transform of the following function of 06  
 $f(x) = 1 - x^2$ , for  $|x| < 1$   
 $= 0$ , for  $|x| > 1$

- b) Obtain Fourier sine Transform of  $\frac{x}{a^2 + x^2}$  and Fourier cosine Transform of  $\frac{1}{a^2 + x^2}$  08

- c) State and prove Parseval's identity in Fourier Transform of  $f(x)$  06

**MODULE - III**

- Q.5 a) Find Laplace Transform of the following functions 08  
 1)  $e^{-3t}(2 \cos 5t - 3 \sin 5t)$       2)  $\frac{1 - \cos t}{t}$

- b) Prove the following 06

i) If  $L[f(t)] = F(s)$ , then  $L[e^{at}f(t)] = F(s - a)$

ii) If  $L[f(t)] = F(s)$ , then  $L[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$

- c) Evaluate the following integral by using Laplace Transform 06

$$\int_0^{\infty} e^{-3t}(t \cos t) dt$$

- Q.6 a) Find Laplace inverse transform of the following functions 08

1)  $\frac{s-2}{(s^2-4s+5)}$       2)  $\frac{1}{s(s^2-a^2)}$

- b) State Convolution theorem and hence find Laplace inverse transform of the following function 06

$$\frac{s^2}{(s^2 + 25)(s^2 + 4)}$$

- c) Solve following initial value problem using Laplace Transforms 06  
 $(D^2 + 9)y = 18t$ ,  $y(0) = 0$ ,  $y'(0) = 0$

## MODULE -IV

- Q.7 a) Derive the solution of one –dimensional wave equation using the method of separation of variables. **08**
- b) By the method of separation of variables solve **12**
- 1)  $\frac{\partial u}{\partial x} + 7 = \frac{\partial u}{\partial t}$  given that  $u(x, 0) = 2e^{-4x}$
  - 2)  $4\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = 0$  given that  $u(x, 0) = e^{-x}$
- Q.8 a) Derive one – dimensional heat equation and state all assumptions made. **10**
- b) A rod 70 cms long has its ends X and Y kept at  $40^\circ$  and  $60^\circ$  respectively until steady state condition prevail. The temperature at each end is then suddenly reduced to  $0^\circ$  and kept so. Find the resulting temperature function  $u(x, t)$  taking  $x=0$  at X. **10**