SE431

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S.E. (Information Technology) (Sem-IV) (Revised Course 2016-2017)
EXAMINATION Nov/Dec 2019
Discrete Mathematical Structures

[Duration : Three Hours] **[Total Marks : 100]**

Instructions:
1) Answer five questions: Two from Part A, two from Part B and one from Part C
2) Assume suitable data if necessary.

PART A

Answer **any two** questions:

1. a) Let A, B and C be subsets of an universal set. Prove that : **(6)**
(i) $P(A \cap B) = P(A) \cap P(B)$ where $P(X)$ denotes power set of X.
(ii) $A - (B \cup C) = (A - B) \cap (A - C)$.

b) Let $S = \{1,2,3\}$ and 'R' be a relation on SxS defined by (a, b) R (c, d) if a + d = b + c. **(7)**
Prove that 'R' is an equivalence relation on SxS and determine all the distinct equivalence classes.

c) Using Mathematical Induction prove that **(7)**
$2^n > n$ for all $n \geq 1$.

2. a) Obtain the principle conjunctive normal form of the following: **(8)**
(i) $(\sim p \vee r) \wedge (p \vee q)$
(ii) $((\sim p \rightarrow r) \wedge (q \rightarrow p)$

b) Let $f: R - \{1\} \rightarrow R - \{3\}$ be given by $f(x) = \frac{3x+4}{x-1}$ **(6)**
Show that 'f' is bijective and hence find f^{-1}.

c) Find the reminder when 16^{53} is divided by 7. **(6)**

3. a) Let $a \equiv b \ (mod\ m)$ and $c \equiv d \ (mod\ m)$. Prove that $a + c \equiv b + d \ (mod\ m)$. **(6)**

b) Show that 'r' is a valid conclusion from the premises $p \rightarrow q, q \rightarrow r, and\ p$. **(8)**

c) Let (A, +, . , -) be a Boolean algebra then prove that **(6)**
(i) $\overline{(a.b)} = \bar{a} + \bar{b}$
(ii) $a + 1 = 1, \quad \forall a, b \in A$.

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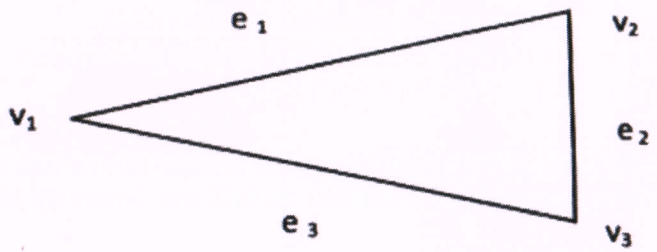
PART B

Answer **any two** questions:

4. a) Solve the following recurrence relation. **(6)**
$a_n + a_{n-1} = 3n \; \forall \, n \geq 1; a_0 = 1.$

b) Suppose there are 200 first-year students; 100 of which are taking calculus and 70 of which are taking algebra. **(8)**
If there are 50 first-year students taking both calculus and algebra, how many first-year students are taking neither course?

c) Define incidence matrix of an undirected graph and represent the following graph by an incidence matrix : **(6)**

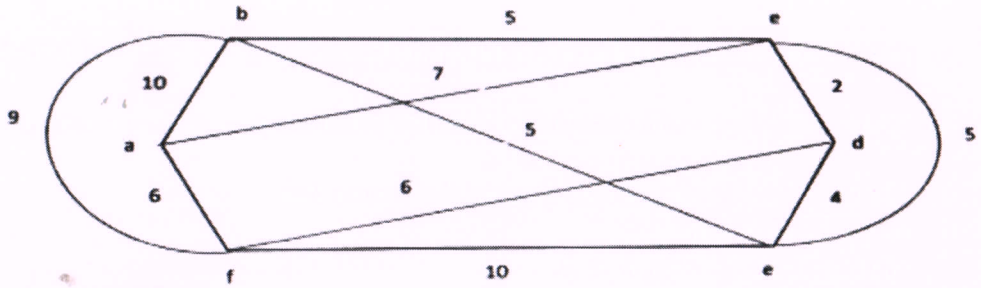


5. a) Find the recurrence relation for the number of binary sequences of length n where the pattern 00 appears for the first time at the end of the sequence. **(6)**

b) Show that among five points placed in a square of side 4cms, there are at least two points at most $2\sqrt{2}$ cms apart. **(8)**

c) Prove that K_5 is not planar. **(6)**

6. a) Determine a minimal spanning tree for the graph given below : **(8)**



b) Using the principle of inclusion and exclusion find the number of positive integers not exceeding 1000 that are divisible by 5 or 7 **(8)**

c) Define a complete bipartite graph. Give an example. **(4)**

PART C

Answer **any one** question:

7.

a) Show that the number of vertices with odd degree in a sample graph is even. **(6)**

b) Let A={2,3,6,12,24,36} and R be the relation of divisibility on A. Construct a Hasse diagram for the poset (A,R) **(6)**

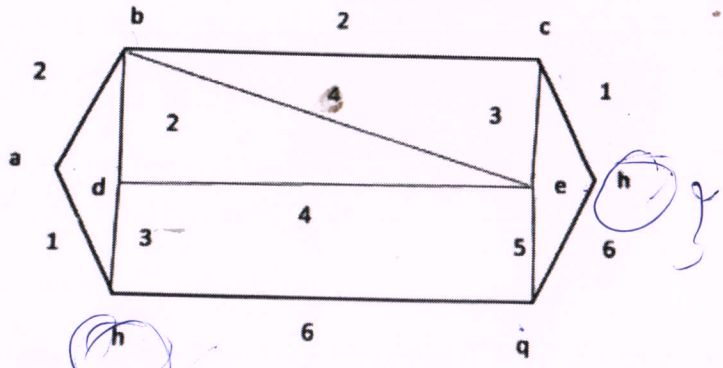
c) i) In how many ways can a cricket team of eleven be chosen from a group of 15 players?
ii) How many teams will include a particular player? **(8)**

8.

a) Let (A, +, . ,-) be a Boolean algebra, then prove that **(6)**
i) $\overline{(a.b)} = \bar{a}\,.\,\bar{b}$
ii) $a.(a+b) = a, \forall a, b \in A.$

b) Show that the set $S = \{\downarrow\}$, where the connective "↓" denotes NOR, is functionally complete. **(6)**

c) Using Dijkstra's algorithm find shortest path between the vertices a and h of the following graph: **(8)**



Please clarify

(2A)