SE413

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S.E. (Electronics & TC / Electronics & Comm Engg) (Sem-IV)
(Revised Course 2016-2017) EXAMINATION Nov/Dec 2019
Probability Theory and Random Processes

[Duration : Three Hours] **[Total Marks : 100]**

Instructions:
1) Attempt **any five** questions, two each from part A and part B and one from part C.
2) Use statistical tables wherever required
3) Assume suitable data, if necessary.

PART - A

Q.1

a) There are two boxes A and B. Box A has3 red and 5 blue balls and box B has 4 red and 3 blue balls. A dice is rolled and if the number appearing on the dice is 1 or 2, a ball is randomly drawn from box A otherwise from box B. Find the probability that the drawn ball is red. If the ball drawn is blue what is the probability that it is from box A. **6**

b) Define independent events. If A & B are events such that P(A) = 0.4, P (B) = 0.3 and $P(A \cap B^c) = 0.28$. Are A and B independent? Justify. **4**

c) Define moment generating function of a random variable. Find the moment generating function of Poisson distribution. **5**

d) The life time of an electric bulb is exponentially random variable with mean of 120 hours. What is the probability that the bulb will last more than 150 hours? **5**

Q.2

a) The time it takes to repair a personal computer is a random variable whose density in hours is uniformly distributed over (0,2). The cost of repair is equal to $40+30\sqrt{t}$ where t is time. Compute the expected cost to repair the computer. **6**

b) State exponential distribution and find its mean. **6**

c) A used car dealership has found that the length of time before a repair is required on the car it sells is normally distributed with mean 10 months and standard deviation of 2 months. What is the probability a car will i) require repairs by the eighth month? ii) Will not need repairs for at least 12 months? If the dealer wants to give guarantee period for free repairs then what should be the period so that atmost 10% cars will need to be repaired. **8**

Q.3

a) The joint density of random variable X and Y is given by **8**

$$f(x,y) = \begin{cases} x+y & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0 & eleswhere \end{cases}$$

Find- i) The marginal density of X ii) Find P (Y<X) iii) E(Y)

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b) When is a random variable said to have lack of memory property? State one standard random variable known to you having this property? Justify. 6

c) A random variable X is uniformly distributed over (0,1) Find the probability density function of $Y=X^2$ 6

PART- B

Q.4

a) In a random sample of 520 families owning television set in Goa, it is found that 358 subscribe to Star Plus. Can one claim that 70% subscribe to star plus? Use LOS of 5%. 7

b) A manufacturer of capacitors claims that the breakdown voltage of his capacitors has a mean value 100. A test of 12 capacitors yielded the following breakdown voltages 96, 98, 102, 92, 111, 110, 99, 103, 95, 101, 106 and 97. Does this validate the manufactures claim? Use level of significance of 5%. 7

c) Given the auto correlation function of a stationary process. 6

$$R_{XX}(\tau) = 22 + \frac{4}{1+6\tau^2}$$

Find its mean and variance.

Q.5

a) Define an evolutionary random process and Ergodic in mean random process. 6

b) Fit a Poisson distribution for the following distribution and also test the goodness of fit. Use Los of 5%. 6

X	0	1	2	3	4	5	6	7
Frequency	314	335	204	806	29	9	3	0

c) A homogeneous Markov chain has three states A, B ,C and it transition Probability matrix is $\begin{bmatrix} .0.4 & .0.6 & .0 \\ .0.3 & .0.2 & .0.5 \\ .0.2 & .0.5 & 0.3 \end{bmatrix}$. If its initial probability vector is (0.5, 0.2, 0.3). 8

Find i) $P(X_2 = B)$ ii) Is the Markov chain regular. If it is find its steady state vector.

Q.6

a) Define wide sense stationary random process. If $X(t) = A\,Cos(w_0 t + \Pi\theta)$, Where A and w_0 are constants and θ is uniformly distributed over (0,2) prove that X(t) is wide sense stationary . 8

b) A random sample of 500 coins has mean weight of 28.55 gms and standard deviation of 1.25 gms. Another random sample of 400 coins has a mean weight of 29.12 gms and standard deviation of 1.4gms. Can the samples be considered to be drawn the same population? 6

c) If customers arrive at a counter in accordance with Poisson process with a mean rate of 4 per minute. Find the probability that the inter arrival time between two arrival is 6

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i) between 1 and 3 minutes ii) 2 minutes or less.

PART – C

Q.7

a) A Binomial random variable X has mean 2 and variance 3/2. Find i) P (X>5) ii) $P\ (2 < X \leq 6)$. **6**

b) Three different washing machines were employed to test four detergents. The following data give a coded score of the effectiveness of each washing. **8**

	Machine		
	1	2	3
Detergent 1	53	50	59
Detergent 2	54	54	60
Detergent 3	56	58	62
Detergent 4	50	45	57

Test the hypothesis that the detergent used does not affect the score. Use Los of 5%.

c) What is hypothesis testing? Explain the two types of errors that arise in hypothesis testing? **6**

Q.8

a) A fair dice is rolled six times Let X be the number of times 1 is rolled and Y be the number of times 6 is rolled. What is the joint probability mass functions of X and Y? Compute the covariance of X and Y? **8**

b) If X and Y are two continuous random variable having joint density f(x,y), define the conditional expectation E(X/Y). show that if X and Y are independent E(X/Y) = E(X) **6**

c) The mean and variance of weight of a sample of 100 men in a city was found to be 67.5 kgs and 8.52kgs respectively. Find i) 95% ii) 99% confidence limits for the mean weight of all men in the city. **6**

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