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S.E. (Electronics & TC / Electronics & Comm Engg) (Sem-IV) (Revised Course 2016-2017)  
 EXAMINATION MAY/JUNE 2019  
 Probability Theory and Random Processes

[Duration : Three Hours]

[Total Marks : 100]

- Instructions:**
1. Attempt any five question, two each from part A and part B and one from part C.
  2. Use statistical tables wherever required.
  3. Assume suitable data, if necessary.

PART A

- Q.1
- a) Three friends and 6 other are randomly seated in a row. What is the probability that the three friends are seated next to each other? 06
  - b) Define independent events. If  $P(A) = 0.25$ ,  $P(B^c \cap A) = 0.15$  and  $P(B) = 0.4$ . Are A and B independent events? 04
  - c) The Phase error X in a tracking device has probability density 06  

$$f(X) = \begin{cases} \cos x & 0 < x < \frac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$
 Find i)  $P(X < \pi/3)$  ii)  $E(X)$  04
  - d) State Binomial distribution and compute its moment generating functions. 04
- Q.2
- a) A random variable X has probability density function  $f_x = \begin{cases} x/2 & 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$  06  
 Find the probability density function of  $Y=X^2$
  - b) State the probability mass function for Poisson random variable with mean  $\lambda$ . Show that the sum of two independent Poisson random variables with means  $\lambda_1, \lambda_2$  is also a Poisson random variable with mean  $\lambda_1 + \lambda_2$  08
  - c) The average daily turnover X (in thousands of Rupees) at a store is 25. The turnover is found to be exponentially distributed. Find i) The probability that the turnover will exceed Rs.30,000. ii) The probability that the turnover will be between Rs.24,000 and Rs.28,000. 06
- Q.3
- a) A two dimensional random variable (X,Y) has probability density function 06  

$$f(x,y) = \begin{cases} 6e^{-2x-3y} & x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases}$$
- Find i) The marginal density function of X & Y ii) Are X and Y independent?
- b) If X and Y are independent continuous random variable. Then show that 06  
 $P(Y \leq X) = \int_{-\infty}^{\infty} F_Y(x) f_x(x) dx$  where  $F_Y$  is the distribution function of Y and  $f_x$  is the density function of X

- c) Define the covariance of two random variables. Find the covariance of random variable (X,Y) 08  
having joint probability density function

$$f(x, y) = \begin{cases} \frac{4}{5}(xy + 1) & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

PART B

- Q.4 a) The joint distribution of marks scored by students in mathematics (X) and physics (Y) is given 08  
by joint density function

$$f(x, y) = \frac{1}{32\pi} e^{-\frac{(x-44)^2 + (y-50)^2}{32}} \quad -\infty < X < \infty, -\infty < y < \infty,$$

Find the probability that a student scores more than 42 marks in mathematics and between 48 and 54 marks in physics.

- b) An IQ test administered to a large group of students yielded a standard deviation of 36. The test 06  
given to a group of 120 boys gave an average of 84, while another group of 125 girls gave an  
average of 89. Does this show that girls have significantly better IQ than boys at a level of  
significance of 5%?
- c) Define a mean-ergodic process. Determine if the constant process  $x(t)=A$ , where A is a random 06  
variable with mean  $\mu_A$  and variance  $\sigma_A^2$  is mean ergodic.
- Q.5 a) A sample poll of 400 votes chosen at random from all votes in a large city indicated that 55% of 06  
them were in favour of a particular political party. Find the i)95% ii) 99% confidence limits for  
the percentage of votes in favour of the party.

- b) The joint density function of X and Y  $f(x, y) = \begin{cases} (y+x) & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$  find the 07  
density function of  $Z=X+Y$

- c) A filling machine is expected to fill 5 kgs of powder into bags. A sample of 10 bags gave the 07  
following weight in 4.7, 4.9, 5.0, 5.1, 5.4, 5.2, 4.6, 5.1, 4.6, 4.7. Test whether the machine is  
working properly. Use Level of significance of 5%.

- Q.6 a) The sales (in thousand of rupees) in a super market during a week are given below. Test the 06  
hypothesis that the sales do not depend on the day of the week, using a level of significance of  
5%.

Days	Mon	Tue	Wed	Thu	Fri	Sat
Sales	143	125	162	136	152	168

- b) Four standard chemical procedures are used to determine the magnesium content in a certain 08  
chemical compound. Each of the procedure is used four times on the given compound with the  
following data resulting.

Method			
1	2	3	4
76.42	80.41	74.2	86.2
78.62	82.26	72.68	84.36
80.40	81.15	78.84	86.04
78.2	79.20	80.32	80.68

Do the procedure indicate that the procedure yield equivalent results at a level of significance of 5%?

- c) Define Poisson Process. Show that the sum of two independent Poisson process is a Poisson process. 06

PART C

- Q.7 a) Define the auto covariance of a random Process  $X(t)$ . If  $X(t) = A \cos t + B \sin t$ , find the auto covariance of  $X(t)$ , were  $A$  and  $B$  are independent random variables.  $A$  having standard normal distribution and  $B$  is uniformly distribution over  $(0,2)$ . 06

- b) When are two random process  $X(t)$  and  $Y(t)$  said to be jointly wide sense stationary? 04

- c) A manufacturer of solar heaters claim that 60% of his heaters work satisfactorily for 10 years. Assuming that the claim is right find the probability that i) 4 of the 6 heater sold will work satisfactorily. ii) 2 out of 5 heaters will not work satisfactorily. 06

- d) Define independent events. If  $A$  and  $B$  are independent prove that  $\bar{A}$  (the complement of  $A$ ) and  $B$  are independent. 04

- Q.8 a) A homogeneous Markov chain has three states  $A, B, C$  and its transition probability matrix is  $\begin{bmatrix} .4 & .3 & .3 \\ .15 & .25 & .6 \\ 0 & .5 & .5 \end{bmatrix}$ . If its initial probability vector is  $(0.5, 0.5, 0)$ . 08

Find i)  $P(X_2 = B)$  ii)  $P(X_0 = A, X_1 = A, X_2 = C, X_3 = B)$  ii) is the Markov chain regular if it is regular find its steady state vector

- b) A Poisson random variable  $X$  satisfies the relation  $P(X=0) + 2P(X=2) = 2P(X=1)$ . Find  $P(X < 3)$ . 06

- c) If customers arrive at a counter in accordance with Poisson process with a mean rate of 3 per minute. Find the probability that the inter arrival time between two arrival is 06

- i) between 1 and 2 minutes ii) more than 2 minutes