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S.E.(Electronics & TC) / Electronics & Comm Engg (Sem-III) (Revised Course 2016-2017)

EXAMINATION MAY/JUNE 2019

Applied Mathematics - III

[Duration : Three Hours]

[Max.Marks : 100]

Instructions:

- 1) Attempt five questions: Any two from Part A, any two from Part B and any one from Part C.
- 2) Make suitable assumptions wherever required.
- 3) Figures to the right indicate full marks.

**Part A**

- Q.1 a) Prove that  $A (adjA) = |A|I$  05
- b) Find the rank of the following matrix. 08
- $$\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$
- c) Find the minimal polynomial of the following matrix. 07
- $$\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$
- Q.2 a) Find the Laplace Transform of the following. 08
- i)  $f(t) = t \cos 2t \cos t$
  - ii)  $f(t) = \sinh \frac{t}{2} \sin \frac{\sqrt{3}}{2} t$
- b) Find the inverse Laplace transform of the following 06
- i)  $F(s) = \frac{e^{-s}}{(s+3)^3}$
  - ii)  $G(s) = \cot^{-1} \frac{s}{2}$
- c) Solve the following differential equation by applying Laplace transforms 06
- $$y''(t) + 4y(t) = \sin \omega t;$$
- With  $y(0) = 0$  and  $y'(0) = 0$
- Q.3 a) Solve the following system of linear equations or indicate the non-existence of solution: 08
- $$\begin{aligned} 4y - 2z &= 2 \\ 6x - 2y + z &= 29 \\ 4x + 8y - 4z &= 24 \end{aligned}$$
- b) Prove that for an orthogonal matrix A,  $|A| = \pm 1$  05

- c) Apply the convolution theorem to compute the inverse Laplace transform of

$$F(s) = \frac{s}{(s^2 + 4)(s^2 + 9)}$$

07

Part B

- Q.4 a) Find the Fourier series expansion of the function  $f(x) = x^2 - x$  in  $(-\pi, \pi)$  with period  $2\pi$ . 07

- b) Find Fourier series for the following function. With period 4 given by:

$$f(x) = x, \quad 0 < x < 2$$

$$= 0, \quad 2 \leq x < 4$$

07

- c) Find the Half range cosine series of  $f(x) = x$  in  $0 < x < 1$

06

- Q.5 a) Evaluate the integral  $\oint_C \frac{z}{(z^2-9)(z+1)}$  where C is the circle  $|z|=2$  07

- b) Expand  $f(z) = \frac{1}{z(1-2)}$  in a Taylor series about  $z=-1$  06

- c) Evaluate the integral  $\oint_C \frac{5z-2}{z(z-1)} dz$  by applying the residue theorem where C is the circle  $|z|=2$ . 07

- Q.6 a) Find the poles of the function  $f(z) = \frac{z^2}{(z-1)^2(z-3)}$  and compute the residue at each pole 08

- b) Evaluate the integral  $\int_0^\infty \frac{1}{x^2-16} dx$  using contour integration 06

- c) Find Fourier series for the following function  $f(x)$  with period  $2\pi$  given by  
 $f(x) = 0, \quad -\pi \leq x < 0$   
 $= x, \quad 0 \leq x < \pi$  06

Part C

- Q.7 a) Verify Cayley Hamilton Theorem for the matrix 06

$$\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$

- b) Let  $L(f(t)) = F(s)$ , where  $F(s)$  is the Laplace Transform of  $f(t)$ . Then prove the following: 08

i)  $L\left(\frac{d}{dt}f(t)\right) = sF(s) - f(0)$

ii)  $L\left(\int_0^t f(u)du\right) = \frac{1}{s}F(s)$

- c) Find the Laplace transform of  $f(t) = t; 0 \leq t < 1$  and  $f(t+1) = f(t); \forall t$  06

- Q.8
- Solve the one dimensional wave equation  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$ . Using the separation of variables method. 08
  - Evaluate the integral  $\int_C \frac{e^z}{z^2 + \pi^2} dz$  where C is the circle given by  $|z|=4$  06
  - Find the Laurent's Series expansion of  $f(z) = \frac{3z-2}{(z+1)(z+2)}$  about  $z=0$ , valid in the region  $1 < |z| < 2$ . 06