

Total No. of Printed Pages:02

S.E. (Electronics & TC/Electronics & Comm Engg) (Semester- III)
(Revised Course 2007-08) EXAMINATION Nov/Dec 2019
Applied Mathematics - III

[Duration : Three Hours]

[Total Marks : 100]

Instructions:

- 1) Answer *any five* questions with *at least one* from each *Module*.
- 2) Assume suitable data if necessary.

MODULE- I

Q.1

a) Show the $\begin{bmatrix} \sin \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan \theta / 2 \\ \tan \theta / 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta / 2 \\ -\tan \theta / 2 & 1 \end{bmatrix}^{-1}$ (5)

b) Find the rank of the matrix by reducing it to its normal form. $\begin{bmatrix} 2 & 1 & 5 & 4 \\ 3 & -2 & 2 & -4 \\ 5 & 8 & -4 & 2 \end{bmatrix}$. (6)

c) Define orthogonal matrix. Prove that the matrix $\frac{1}{3} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal. (5)

d) If a square matrix A is orthogonal prove that $\det(A) = \pm 1$. (4)

Q.2

a) Are the following vectors (1,3,2); (5,-2,1); (-7, 13, 4) linearly dependent? If so find the relation between them. (6)

b) Determine for what value of λ and μ the following system of equations $x+y+z=6$; $x+2y+3z=10$, $x+2y+\lambda z = \mu$ has a i) no solution ii) unique solution iii) more than one solution. (8)

c) Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ using elementary transformation. (6)

Module - II

Q.3

a) Diagonalize the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Hence find A^5 . (8)

b) If $\lambda \neq 0$ is the eigen value of a square matrix A then show that $\frac{|A|}{\lambda}$ is the eigen value of $\text{adj } A$. (5)

c) Verify Cayley -Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and use it to find its inverse. (7)

Q.4

a) If $f(x) = \begin{cases} \pi x & \text{if } 0 \leq x < 1 \\ \pi(2-x) & \text{if } 1 \leq x \leq 2 \end{cases}$. Find the Fourier series of $f(x)$ in the range (0, 2). (8)

Deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

- b) Find the half range Cosine series of $f(x) = x(1-x)$ for $0 \leq x \leq 1$. (6)
- c) Define an orthogonal set of functions. Prove that the set $\left\{ \cos \frac{n\pi x}{2} \right\}$ $n = 0, 1, 2, \dots$ is orthogonal over $[-2, 2]$. (6)

Module – III

- Q.5** a) If $L(f(t)) = F(s)$, where $L(f(t))$ denotes the Laplace transform of $f(t)$, prove the following (6)
- i) $L(e^{at} f(t)) = F(s-a)$ ii) $L\left(\int_0^t f(t) dt\right) = \frac{1}{s} F(s)$ (6)
- b) Find the Laplace transform of (6)
- i) $t e^{-t} \sin 2t$ ii) $(e^{2t} - \cos 2t)/t$
- c) Solve the ordinary differential equation, using Laplace transforms (8)
- $$y''(t) + 3y'(t) - 4y(t) = e^{2t}, y(0) = 1, y'(0) = 0$$

- Q.6** a) If $f(t)$ is a periodic function having period p , then prove that (9)
- $$L(f(t)) = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t) dt$$
- Find the Laplace transform of $f(t) = 3t+1$ $0 < t < 2$, $f(t+2) = f(t)$
- b) Solve the integro – differential equation using Laplace transform (6)
- $$\frac{dy}{dt} + \int_0^t y(t-u)e^u du = 2e^{2u} \quad y(0) = 0$$
- c) State convolution theorem. Use it to find Laplace transform of $\int_0^t \cos(t-u)e^{2u} du$ (5)

Module – IV

- Q.7** a) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ (8)
- Hence evaluate the integral $\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$
- b) Define convolution of two functions. Find the convolution of $f(t) = t$, $g(t) = e^{-t}$ for $t \geq 0$ and $f(t) = g(t) = 0$ for $t < 0$. (6)
- c) Find the Fourier Sine transform of $f(x) = e^{-2x}$. (6)

- Q.8** a) Find the Z- Transform of i) $2n + \frac{1}{n!}$ ii) $\cos(2\pi n) + 5.2^n$ (6)
- b) Solve the difference equation $y_{n+2} - 4y_{n+1} + 3y_n = 1$; given $y_0 = 0$ and $y_1 = 1$. (8)
- c) State the one dimensional heat equation and derive its general solution. (6)