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S.E. Computer (Sem-III) (Revised Course 2016-2017) EXAMINATION MAY/JUNE 2019
Applied Mathematics - III

[Duration : Three Hours]

[Max.Marks : 100]

Instructions:

- 1) Attempt five questions: Any two from Part A, any two from Part B and any one from Part C.
- 2) Make suitable assumptions wherever required.
- 3) Figures to the right indicate full marks.
- 4) Statistical Tables will be provided.

Part A

- Q.1 a) 06
- i) Prove that for any two orthogonal matrices A and B, AB is an orthogonal matrix.
 - ii) If A and B are symmetric matrices. Prove that AB-BA is a skew symmetric matrix.
- b) Convert to normal form and hence find the rank of the matrix. 07
- $$\begin{pmatrix} 3 & 4 & 1 & 1 \\ 2 & 4 & 3 & 6 \\ -1 & -2 & 6 & 4 \\ 1 & -1 & 2 & -3 \end{pmatrix}$$
- c) The mean height of 500 male students in a certain college is 151 cm and the standard deviation is 15cm. Assuming the heights are normally distributed, find how many students have heights between 120 and 155cm. 07
- Q.2 a) Find a matrix P which transforms the following Matrix to Diagonal form and hence find A^4 : 07
- $$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$
- b) Determine the values of λ and μ for the which the following system of equations has 07
- a. Unique solution
 - b. No solution
 - C. Infinite solution
- $$2x + 3y + 5z = 9; 7x + 3y - 2z = 8; 2x + 3y + \lambda z = \mu$$
- c) An electronic component consists of three parts. Each part has probability 0.99 of performing satisfactorily. The component fails if 2 or more parts do not perform satisfactorily. Assuming that the parts perform independently. Determine the probability that the component does not perform satisfactorily. 06
- Q.3 a) Find the correlation coefficient and the equations of regression lines from the following data. 08
- | | | | | | |
|---|---|---|---|---|---|
| x | 1 | 4 | 2 | 3 | 5 |
| y | 3 | 1 | 2 | 5 | 4 |

- b) The amount of time that a watch will run without having to be reset is a random variable having an exponential distribution with mean 120 days. Find the probability that such a watch will
- Have to be set in less than 24 day
 - Not have to be reset in at least 180 days
- c) Examine the following system of vectors for linear dependence. If dependent, find the relation between them
 $X_1 = (1, -1, 1); X_2 = (2, 1, 1); X_3 = (3, 0, 2)$

Part B

- Q.4 a) Find the Laplace Transform of
- $\int_0^t \sin t e^{-t} dt$
 - $\frac{1 - \cos t}{t}$
- b) Evaluate the following Integrals
- $\int_0^{\infty} e^{-6t} \cos^2 t dt$
 - $\int_0^{\infty} e^{-t} t^2 \sin t dt$
- c) Find the Fourier Transform of
 $f(x) = 1 - |x|$ for $|x| < 1$
 $= 0$ for $|x| > 1$
 and hence find the value of
 $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt$
- Q.5 a) Find the Fourier Cosine Transform of e^{-ax} and hence Find Fourier sine of $\frac{e^{-ax}}{x}$
- b) Find that the inverse Z transform of
 $\frac{z}{z^2 + 7z + 10}$
- c) If F(s) is the Fourier transform of f(x), then
- $F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$
 - $F\{f(x - a)\} = e^{isa} F(s)$
- Q.6 a) Find the Inverse Laplace Transform of
- $\frac{s}{(s+1)(s^2+2s+6)}$
 - $\frac{s}{(s^2+1)(s-1)}$
- b) State and prove the convolution theorem for Laplace Transforms.
- c) Solve the following differential equations using Laplace Transforms:
 $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = e^{-2t}$ given that $y(0) = \frac{dy(0)}{dx} = 0$;

Part C

- Q.7 a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. **08**
- b) Find the Laplace inverse transform of
- i) $\log\left(\frac{\omega^2}{s^2} + 1\right)$ **06**
- ii) $\tan^{-1}(s+1)$ **06**
- c) Find the Fourier cosine transform of $e^{-a^2x^2}$ **06**
- Q.8 a) The mean and variance of a Binomial distribution are 4 and 3. Find
- i) $P(X \leq 2)$ **06**
- ii) $P(X \geq 3)$ **06**
- b) Verify Cayley Hamilton theorem for the following matrix and hence find the inverse of A: **06**
- $$A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \\ 4 & -2 & 1 \end{pmatrix}$$
- c) Solve $\frac{dx}{dt} - y = e^t$; $\frac{dy}{dt} + x = \sin t$ given $x(0) = 1$; $y(0) = 0$ **08**