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S.E. (Computer) (Semester- III) (Revised Course 2007-08)
EXAMINATION NOV/DEC 2019
Applied Mathematics - III

[Duration : Three Hours]

[Total Marks : 100]

Instructions:

- 1) Attempt any five questions and atleast one from each module.
- 2) Assume suitable data, if necessary.
- 3) Figures to the right indicate full marks.
- 4) Use statistical tables wherever required.

Module-I

1 a) Define Linearly Dependent vectors. (7)

Test whether the following vectors are linearly independent or not.

$$X_1 = (2 \ 2 \ 7 \ -1) \ X_2 = (3 \ -1 \ 2 \ 4) \ \text{and} \ X_3 = (1 \ 1 \ 3 \ 1)$$

b) Express the following matrix as the sum of a symmetric and a skew symmetric matrix. (6)

$$A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$$

c) Test for the consistency and solve if consistent the following system of equations. (7)

$$3x + 2y + 3z = 1$$

$$x - y + 2z = 2$$

$$x + 4y - z = -3$$

$$-2x - 3y - z = 1$$

2 a) Find the rank of the matrix by reducing it to normal form (7)

$$A = \begin{bmatrix} 2 & 1 & 5 & 4 \\ 3 & -2 & 2 & -4 \\ 5 & 8 & -4 & 2 \end{bmatrix}$$

b) Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ Verify Cayley Hamilton Theorem for A and hence show that (7)

$$A^{-1} = A^3$$

c) Find the eigen value and eigen vector of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ (6)

Module-II

- 3 a) The following marks have been obtained by a class of students in statistics. (8)

Paper I	80	45	55	56	58	60	65	68	70	75	85
Paper II	81	56	50	48	60	62	64	65	70	74	90

Compute the coefficient of correlation for the data given above. Also find the lines of regression.

- b) In a random experiment $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{3}$, $P(B/A) = \frac{3}{4}$, Find $P(A \cup B)$
- c) Let X be a random variable with density function given by
- $$f(x) = \begin{cases} kx^2 & -3 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$
- Determine the value of k and $P(1 \leq X \leq 2)$. (6)

- 4 a) The probability that an individual suffers a bad reaction after taking a certain drug is 0.002. Determine the probability that out of 1000 individuals,
- exactly 3 individuals will suffers a bad reaction.
 - more than 2 individuals will suffer a bad reaction.
- b) Find the moment generating function for the binomial distribution. (4)
- c) What is the probability that a leap year, selected at random, will contain 53 Sundays? (4)
- d) If A and B are independent events, show that A^c and B^c are also independent. (6)

Module-III

- 5 a) Find the Laplace Transform of the following. (7)
- $\frac{1-\cos 2t}{t}$
 - $\int_0^t t^2 e^{-2t} dt$
- b) If f(t) is a continuous periodic function having period T, then prove that (6)
- $$L[f(t)] = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$$
- c) Using Convolution Theorem, find the inverse Laplace transform of $F(S) = \frac{3s}{(s-3)(s^2+1)}$ (7)
- 6 a) If $L[f(t)] = F(s)$, where L denotes the Laplace transform. Prove that (6)
- $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s) ds$
 - $L[tf(t)] = -\frac{d}{ds}[F(s)]$
- b) Find the inverse Laplace Transform of (7)
- $\log\left(1 + \frac{1}{s^2}\right)$
 - $\frac{s}{(s-1)(s^2+1)}$
- c) Use Laplace Transform to solve $y'' - 3y' - 4y = 2e^{-t}$, $y(0) = 0 = y'(0)$ (7)

Module- IV

- 7
- a) Find the Fourier Transform of $f(x) = e^{-4x^2}$ (7)
- b) If $F[f(x)] = F(s)$ is the Fourier transform of $f(x)$, show that (6)
- i) $F[f(x - a)] = e^{ias} F(s)$ ii) $F[f'(x)] = -i s F(s)$ if $f(x) \rightarrow 0$ as $x \rightarrow \infty$
- $f(x) = x$ if $0 < x < 1$
- c) Find the Fourier Sine Transform of $f(x) = 2 - x$ if $1 < x < 2$ (7)
- $= 0$ if $x > 2$
-
- 8
- a) Find the Z-transform of the following (7)
- i) $\frac{1}{n}$ ii) $\cos\left(\frac{n\pi}{2}\right)$
- b) Find the inverse Z-Transform of $\frac{2z^2 - z}{(z-1)(z+2)^2}$ (6)
- c) Solve the difference equation $y_{n+2} + 5y_{n+1} + 4y_n = 2^n$ using Z-Transform method, (7)
- given $y_0 = 0, y_1 = 1$.