Paper / Subject Code: SE370 / Applied Mathematics - III

SE370

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S.E (Computer) Semester-III (Revised Course 2007-08)
EXAMINATION Aug/Sept 2019
Applied mathematics-III

[Duration : Three Hours] **[Max. Marks : 100]**

Instructions:-

1) Answer five questions and at least one from each module
2) Statistical tables are allowed.
3) Make suitable assumptions wherever required
4) Figures to the right indicate full marks.

MODULE –I

Q.1 a) Find the rank of the following matrix: 6

$$A = \begin{pmatrix} 1 & 1 & -3 & -1 \\ 4 & -2 & 6 & 8 \\ 15 & -3 & 9 & 21 \end{pmatrix}$$

b) Test whether the vectors $X_1 = (1,\ 2, -3,\ 4); X_2 = (3, -1,\ 2,\ 1)$ *and* $X_3 = (1, -5, 8, -7)$ are linearly independent. 6

c) Test the consistency of the following equations and if possible find a solution 8

$$4x - 2y + 6z = 8;$$
$$x + y - 3z = -1 \text{ and}$$
$$15x - 3y + 9z = 21$$

Q.2 a) Find the eigen values and eigen vectors of $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ 8

b) Verify the Cayley – Hamilton theorem for the following matrix 8

$$A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

c) Prove that, if A and B are similar matrices then they have the same eigen values. 4

MODULE - II

Q.3 a) If $P((A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{2}$, find $P(A)$ and $P(B)$ **6**

b) A continuous random variable X has a density function given by, **6**

$$f(x) = k(x+1), 2 < x < 5$$
$$= 0, \quad elsewhere$$

find the value of k and hence compute $P(X < 4)$

c) Find the mean and variance of the Uniform Distribution on [0,1] **8**

Q.4 a) A certain type of storage battery lasts on the average 3.0 years with standard deviation of 0.5 years. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 2.3 years. **6**

b) Calculate the correlation coefficient for the following heights (in inches) of fathers and their sons: **8**

Father (X)	65	66	67	67	68	69	70	72
Son (Y)	67	68	65	68	72	72	69	71

c) A manufacturer claims that at least 95 per cent of the equipment which he supplies to a factory is as per the specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test the claim of the manufacturer. **6**

MODULE –III

Q.5 a) Prove that $L[f^{(1)}(t)] = sF(s) - f(0)$, where $L[f(t)]$ denotes the Laplace transform of f(t). **4**

b) Find the Laplace transform of **8**

i) $f(t) = (1 + te^t)^3$ ii) $g(t) = e^{-2t}\cosh 2t$

c) Find the Laplace transform of the following function: **8**

$$f(t) = \begin{cases} \sin\omega t; & for\ 0 \le t < \frac{\pi}{\omega} \\ 0 & ; for\ \frac{\pi}{\omega} \le t < \frac{2\pi}{\omega} \end{cases} \quad with\ f\left(t + \frac{2\pi}{\omega}\right) = f(t)$$

Q.6 a) Apply the convolution theorem, to find the inverse Laplace transform of the function 6

$$F(S) = \frac{1}{(s+2)(s+5)}$$

b) Find the inverse Laplace transform of the following functions: 8

i) $F(s) = \frac{s}{(S+2)^2+4}$

ii) $G(s) = \frac{S+3}{S^2+6S+13}$

c) Solve $\frac{dy}{dt} + 4y + 5\int_0^t y\,dt = e^{-t}$ $given\ y(0) = 0,$ 6

Using Laplace transform.

MODULE- IV

Q.7 a) Find the Fourier transform of $(x) = \begin{cases}(1-x^2); if\ |x| < 1 \\ 0; \quad if\ |x| > 1\end{cases}$ 7

b) Find Fourier cosine transform of $f(x) = e^{-kx}, wherer\ x > 0\ and\ k > 0.$ 7

c) If $F[f(x)] = F(s), then\ prove\ that\ F\{x^n f(x)\} = (-i)^n \frac{d^n}{ds^n} F(s)$ 6

Where $F[f(x)]$ denotes the Fourier transform of f(x)

Q.8 a) If $z[f(n)] = F(z)$, then prove that 8

i) $z[a^n f(n)] = F\left(\frac{z}{a}\right)$

ii) $z[nf(n)] = (-z)\frac{dF(z)}{dz}$

Where Z [f(n)] denotes the Z- transform of f(n).

b) Find the inverse Z- transform of the following function using the convolution theorem 6

$$F(z) = \frac{8z^2}{(2z-1)(4z-1)}$$

c) Solve $y_{n+2} - 4y_n = 2^n, given\ y_o = 0\ and\ y_1 = 0\ , using\ z - transform.$ 6